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RESEARCH DEPARTMENT

The K-rating of lenses

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THE K-RATING OF LENSES

K-rating provides a useful measure of the technical quality of a television apparatus or link and is widely used for performance specification and testing. The optical quality of lenses, on the other hand, is described by Figures of Merit; these being obtained by a procedure somewhat different to that used in K-rating.

In this report, the two quantities are examined and some of their merits and limitations are outlined. The Figure of Merit has two components, and one of these, a measure of lens resolution, is shown to be mathematically related to K-rating. A formula expressing this relationship is determined. Use of this formula will enable the image-resolving properties of lenses to be compared with that of the other elements in the signal chain.

The purpose of the work described was to ascertain whether lens specifications should include a K-rating, but in view of the relationship mentioned above it was decided that the inclusion of a specific measure of K-rating would not be necessary because such information is implicit in the currently used Figure of Merit.

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THE K-RATING OF LENSES

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THE K-RATING OF LENSES

SUMMARY

The subjective quality of lenses and the performance of television apparatus may be described by means of Figures of Merit and K-ratings respectively; in this report factors influencing these quantities are examined. The K-ratings of a number of lenses are worked out and it is found that a relationship exists between the K-rating and one of the two quantities determining Figure of Merit; a formula expressing this relationship is derived. Finally the methods by which the K-rating representing the resolution of a lens may be obtained in practice are considered, and recommendations are made.

1. INTRODUCTION

The subjective quality of a lens may be described by its "Figure of Merit".[1] This figure expresses the deterioration in television-picture quality produced by the lens in terms of "liminal units" - one limen implying that 50% of observers can perceive an impairment to the picture whilst the other 50% are unaware that the impairment is present. Figures of Merit may also be derived from the results of objective measurements by the addition of two quantities, one a measure of the resolution of the lens, the other a measure of its vignetting characteristic.

The quality of electronic apparatus used in television may be described by what is termed its "K-rating".[2,3] This quantity is derived from results of objective measurements; in its most accurate form it is principally obtained from the time-response of the apparatus to a unit impulse band-limited by the system in use.

Figures of Merit and K-ratings are no more than tentative and therefore approximate measures of performance; nevertheless they respectively enable comparisons to be made between different lenses and between different types of apparatus, and the results of such comparisons are in fair agreement with opinions resulting from practical experience.

Since every complete television process involves the use of at least one lens, it would be valuable to compare the subjective quality of lenses with that of the other elements in the signal chain. To do this it is necessary for the same performance criteria to be applied to both. This report considers how the K-rating criteria may be applied to a restricted degree in the assessment of lenses.

2. FIGURE OF MERIT AND K-RATING

In this section the methods for obtaining Figures of Merit and K-ratings are briefly outlined and factors influencing the two quantities are considered. Emphasis is placed on those aspects which are relevant to the application of K-rating to lenses. More comprehensive descriptions of the two techniques may be found in References 1, 2 and 3.

2.1. Figures of Merit

Measurements of lens resolution may conveniently be made by causing the lens to image a narrow slit, and examining the "spread function" of intensity produced at the image. The spatial-frequency/response, termed the "optical transfer-function", is then obtained as the Fourier transform of the spread function. The process is applied to images on axis and at (say) three different positions within the field of the lens, measurements being made in both the sagittal and tangential directions. The moduli of the complex optical transfer-functions, known as "modulation transfer-functions", are then plotted.

A "sharpness factor" is obtained from each modulation transfer-function by dividing the area under the curve taken as far as the cut-off frequency of the system by the area under the rectangular characteristic corresponding to uniform response within the system bandwidth and zero response elsewhere. The sharpness factors are then modified by a series of weighting factors. These allow for the fact that the area of picture subtended by a small change in field angle varies with field angle, and for the reduced "interest value" toward the edges of the picture as compared with that at the centre. This leads to a value of "integrated sharpness", s, which is a measure of the resolution of the lens considered over the whole picture area.

The impairment caused by lack of sharpness is then expressed as:

$$L_S = -9{\cdot}2\ (1 - s)\ \text{liminal units*}$$

Vignetting effects are excluded from the above procedure, because each modulation transfer-function is normalized so as to make its magnitude equal to unity at zero spatial frequency. Vignetting is therefore measured in a separate experiment and the impairment which it causes is expressed as:

$$L_V = -3{\cdot}3\ (\bar{1}{\cdot}8 - \log_{10}V)\ \text{liminal units for } \log_{10}V < \bar{1}{\cdot}8$$

or $$= 0 \text{ for } \log_{10}V \geqslant \bar{1}{\cdot}8$$

where V is the ratio of the intensity at the periphery of the field to that at the centre.

The Figure of Merit of the lens, taking into account both lack of sharpness and vignetting is then:

$$L = (L_S + L_V)$$

---

* This relationship was determined[4] using an electrical analogue that degraded picture definition in the horizontal direction only, although lenses degrade definition in both directions. The error caused by this discrepancy may be significant.

2.2. K-rating

There are two methods by which the K-rating of apparatus may be determined. One method is used for acceptance tests and principally involves a computation of the time response of the apparatus to the signal obtained by passing a unit impulse through an ideal low-pass filter (i.e. to a $(\sin 2\pi f_c t)/2\pi f_c t$ pulse having zeros at $\pm 1/2f_c$, $\pm 1/f_c$, $\pm 3/2f_c$, etc., where f_c is the bandwidth of the (low-pass) system); the spectrum of such a signal is continuous and constant as far as f_c and is zero thereafter. The apparatus under test will, in general, have an imperfect response and its output will therefore be a distorted version of the input pulse. Fig. 1 shows, at (*a*), the waveform of a band-limited impulse that is regarded as the input to the apparatus and, at (*b*), a typical output waveform. It will be seen that the output has magnitudes B_1, B_2, etc., at times when it should be passing through zero. It is from the series of values $B(rT) = B_{-9}, B_{-8}, \dots B_{-r}, \dots B_{-1}, B_0, B_1, \dots B_r, \dots B_8, B_9$, normalized so that $B_0 = 1$, that the K-rating is deduced.

A further series $C(rT) = C_{-8}, C_{-7}, \dots C_{-1}, C_0, C_{+1}, \dots C_7, C_8$ is obtained as the serial product of $B(rT)$ and the series $[\tfrac{1}{2}\ 1\ \tfrac{1}{2}]$, i.e.

$$C_r = \tfrac{1}{2} B_{r-1} + B_r + \tfrac{1}{2} B_{r+1}$$

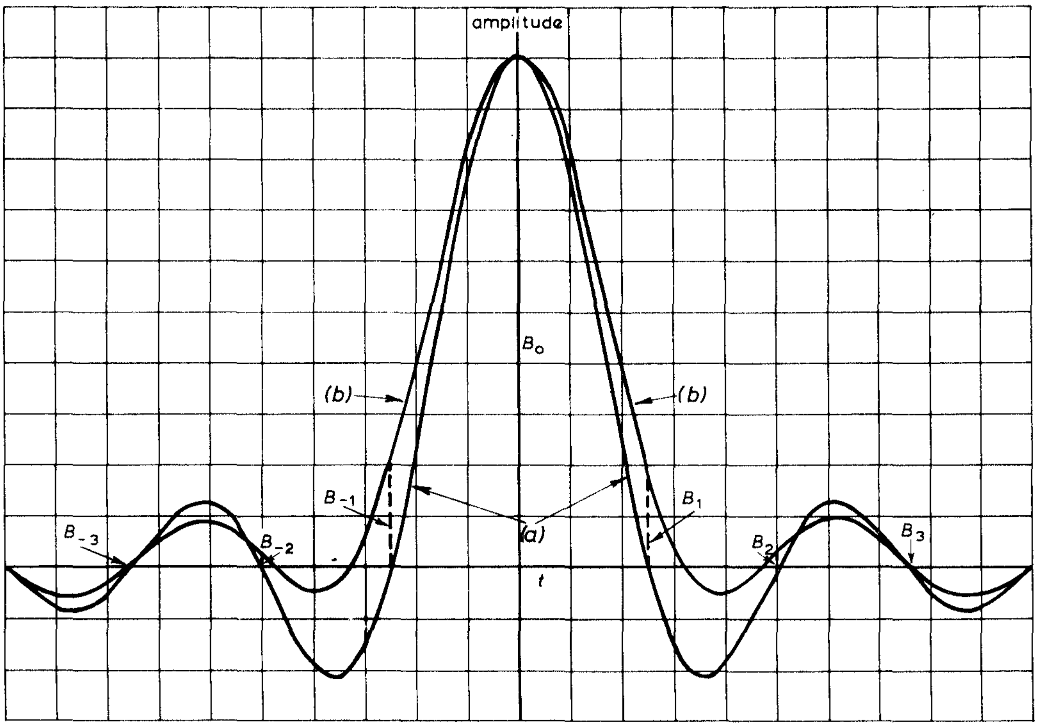

Fig. 1 - Distortion by apparatus of band-limited impulse waveform
(*a*) Band limited impulse (*b*) Response of apparatus to (*a*)

Four restrictions are then imposed:

(i) $$\frac{1}{8}\left|\frac{C_r}{C_o} - \tfrac{1}{2}\right| \leqslant K_1 \qquad r = \pm 1$$

and $$\frac{1}{8}\left|r\frac{C_r}{C_o}\right| \leqslant K_1 \qquad \left\{\begin{array}{l} -8 \leqslant r \leqslant -2 \\ +2 \leqslant r \leqslant +8 \end{array}\right.$$

and $$\left|\frac{C_r}{C_o}\right| \leqslant K_1 \qquad \left\{\begin{array}{l} r < -8 \\ +8 < r \end{array}\right.$$

(ii) $$\tfrac{1}{4}\left|\frac{1}{C_o}\left(\sum_{-8}^{+8} B_r\right) -1\right| \leqslant K_2$$

(iii) $$\frac{1}{6}\left|\sum_{-8}^{+8} B_r -1\right| \leqslant K_3$$

(iv) $$\frac{1}{20}\left(\sum_{-8}^{+8} |B_r| -1\right) \leqslant K_4$$

In practice it is not feasible to attempt an approximation to a band-limited impulse for testing purposes. Instead a 1T, sine-squared, pulse is used,[2] and corrections are applied to the measured output waveform[5] to enable the response to a band-limited impulse to be deduced.

Half-line-duration "bar" and 50 c/s square wave signals are also applied to the apparatus during acceptance testing. The output is examined using an oscilloscope fitted with a calibrated mask in place of the normal graticule; the mask shows limits within which the waveform should lie if specific rating factors are to be met.

The highest figure that satisfies all of the above conditions is then chosen as the K-rating of the apparatus.

The second method of K-rating is used for routine testing and gives an approximate figure. The bar and 50 c/s square wave signals are applied as already outlined but a 2T pulse is used in place of the 1T pulse; the 2T pulse has a spectrum contained almost entirely within the system bandwidth. The oscilloscope mask used in this case also shows the limits within which the response to the 2T pulse should lie for specific K-rating factors to be met. Thus, in this method the K-rating of the apparatus is directly obtained from the extent to which the shape of the output waveform deviates from that of the given input waveform.

2.3. General Observations

In order that a direct equivalence between Figure of Merit and K-rating could exist, two conditions need to be met. The two quantities should take account of similar types of impairment and in so doing they should give an equally reliable measure of picture quality. The implications of these requirements will now be examined. It will be shown that both quantities are to some extent deficient, in that neither was formulated so as to take account of all the picture impairments that can occur in the circumstances in which it is applied. Nevertheless some common ground does exist and therefore it should be possible to determine a direct equivalence under certain rather limited conditions.

The Figure of Merit of the lens takes into account most of its geometrical aberrations but does not include the effects of geometrical distortion or of veiling glare. Geometrical distortion is generally small for fixed focus lenses.* Where there is an abnormal degree of veiling glare, however, the Figure of Merit is an optimistic measure of picture quality. The condition within electronic apparatus which most nearly approximates to that of veiling glare in a lens is an undue emphasis of the d.c. component. This component would be ignored during a measurement of K-rating and an optimistic assessment would again result.

From Sections 2.1 and 2.2 it will be appreciated that L_S, which is obtained from the sharpness factor and contributes to the Figure of Merit, is similar to a K-rating, in that both are based on the frequency/response characteristic. It is therefore simple, in principle, to apply K-rating criteria to the measurement of lens resolution. However, lenses have two properties which influence the Figure of Merit but which have no counterpart in K-rating. These properties will now be discussed.

Apart from certain types of shading in camera tubes, no effect comparable to vignetting occurs in electronic apparatus,** and the K-rating technique is not adaptable to the measurement of such effects. However, as Section 2.1 has indicated, the degradation caused by vignetting is considered separately to that due to lack of sharpness, the two indices L_S and L_V being subsequently added together. It is therefore considered permissible to ignore vignetting whilst applying the K-rating criteria to the measurement of lens resolution; it appears unjustifiable, however, to assume that the formula established relating the sharpness index L_S with K-rating could be used to convert the complete Figure of Merit ($L_S + L_V$) into an associated K-rating.

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* Zoom lenses may exhibit considerable geometrical distortion.

** It is important here to distinguish between the effects of lens vignetting and of low-frequency distortion in electronic apparatus. The former is a function of position within the picture but not of spatial frequency. The latter is a function of spatial frequency but not of position.

The K-rating technique does not deal with variations in resolution as a function of position in the picture; nevertheless it is considered permissible to establish the relationship between L_s and K by considering "ideal" lenses that have equal resolution over the whole field and then to use it to find a K-rating for real lenses whose resolution varies over the field (i.e. where the values of L_s were computed using an integrated sharpness factor).

Since the resolution of lenses varies with position in the field, a given optical transfer-function will, in general, only apply within a small region surrounding the point at which the measurement is made; such a region is termed an "isoplanatism patch". However if, ignoring the effects of vignetting, each isoplanatism patch were in turn enlarged to a size sufficient for the portrayal of low-spatial-frequency picture information, the attenuation of such information would in all cases be very small. Therefore when determining the contribution made by the resolution of the lens to K-rating, it is not necessary to consider the reproduction of half picture-width and half picture-height "bar" patterns; the distortion of such patterns would almost entirely be due to vignetting.

Before making the calculations to be described, an attempt was made to predict the relationship between L_s and K-rating on the basis of current experience. This proved quite difficult, however. On the one hand, the magnitude of L_s is not obviously related to the grading on the six-point quality scale; one might expect -1 liminal units to describe distortions which are graded 1½ on a quality scale,* whilst for a lens whose quality in television is described as "rather poor" (grade 4) the value of L_s might be about -3½ liminal units.

On the other hand, the way in which K-ratings are related to subjective impression has not been established with certainty. A useful guide, however, has been provided by Newell and Taylor[6], who made measurements of the waveform distortion of signals received during the 1957/8 UHF field trials and compared the K-ratings obtained with the results of a subjective assessment of picture quality. Unfortunately the picture sources used were often of inferior quality, and so the picture was not rated better than grade 2 even when the propagation path was extremely good. Fig. 2 is a reproduction of the graph given in Reference 6, to which a suggested correction allowing for picture source impairment has been added.

It is not possible, then, to predict with confidence the relationship between L_s and K-rating. Fig. 3 shows a tentative scale derived from the above information.

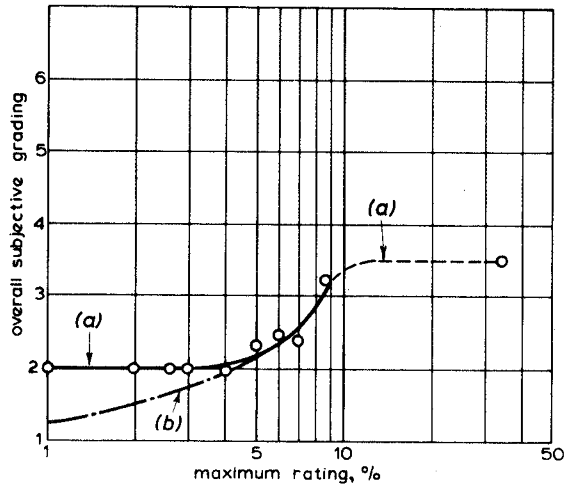

Fig. 2 - Relationships between K-rating and 6-point quality scale

——— Results of measurements by Newell and Taylor

—·—·— Proposed correction to the above curve to remove the effect of impairments at the source of the transmitted picture.

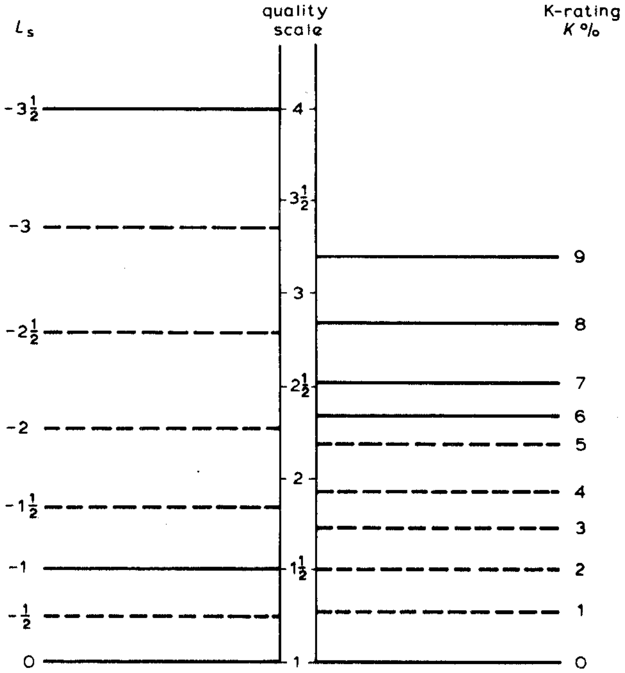

Fig. 3 - Tentative scale relating L_s and K-rating

——— substantiated — — — interpolated

3. DETERMINATION OF THE RELATIONSHIP BETWEEN L_s AND K-RATING

This section describes two methods used to establish the relationship between L_s and K-rating. The first method was experimental, making use of data obtained during normal lens measurements. The second method involved a purely theoretical approach. It arose out of the first, when it was found that of the four restrictions defining K-rating, and described in Section 2.2, the third was always dominant. This led to an exact relationship between K-rating and the index L_S derived from the sharpness factor.

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* Assuming, for simplicity, that 50% of observers vote grade 1 (excellent) and the other 50% vote grade 2 (good).

Section 2.2 has indicated that, neglecting vignetting effects, the K-rating of a lens involves its response to a spatial (sin x)/x function of intensity, x being a unit of length. Such a distribution could never be achieved in practice, if only because negative intensity has no physical meaning. The function is therefore an unreal one, in the same way that the C.I.E. colour primaries are unreal. Nevertheless as a mathematical artifice it may conveniently be used to determine K-rating, just as the unreal primaries lead to a convenient expression of chromaticity.

3.1. The Experimental Approach

The relationship between L_S and K-rating was first determined by applying the K-rating method to a number of lenses whose values of L_S were determined from measurements of their frequency/response characteristics in the way described in Section 2.1.

The results of many previous measurements were available, and those relating to three vidicon lenses were used. These measurements had been made on a modified Rank Taylor Hobson Mark I Optical Test Bench[1] and so both spread functions and optical transfer-functions were available.

In fact all but one of the lenses considered in the analysis were fictitious. Some of the measurements whose results were used, for instance, related to the 12 mm Vidital (No. 570897) at an aperture of T2. For a lens having, over its whole field, the resolution of the above lens measured at 30 °T (tangential) the value of L_S would be equal to -1·99 limens; for one which had, over its whole field, the resolution measured at 30 °S (sagittal) the value of L_S would be equal to -0·94 limens. A lens exhibiting over its whole field a spread function similar to that measured at 30 °S but somewhat expanded (the distance scale being reduced by the factor 0·91) would be characterised by a value of L_S equal to -1·08 limens. As explained in Section 2.3, the relationship between L_S and K-rating is assumed to be universally valid even if it is determined using lenses whose response is considered to be invariable with angle; moreover, the calculation is considerably simplified if such lenses are considered. Twenty-four hypothetical lenses of this type were therefore used. These exhibited a wide variety of spread functions and had a representative range of optical transfer-functions; the values of L_S lay between -0·48 and -3·16 limens, a range covering most television lenses. In addition the K-rating of one real lens, the 12 mm Vidital, was computed. In order to enable this to be done the optical transfer-functions obtained at various angles were combined using the weighting factors mentioned in Section 2.1 to produce a "composite" frequency/response characteristic. A composite spread-function was similarly obtained from the individual spread-functions.

The intensity response of these lenses to a spatial (sin x)/x intensity distribution can be determined by a number of methods. A preliminary computation was therefore made in which three methods were examined, using data relating to three of the hypothetical lenses previously referred to together with the composite data relating to the real lens. The three methods will be described in ensuing Sections.

The (sin x)/x distribution used was appropriate to a cut-off frequency of 20 c/mm, which corresponds to 3 Mc/s at the 405-line standard on a vidicon format (i.e. the spacing between zeros was 25 μ).

3.1.1. The Convolution Method

The spatial response of the lens to a (sin x)/x intensity distribution may be obtained by convolving the (sin x)/x function with the spread function of the lens. Samples of the (sin x)/x distribution were therefore taken at 5 μ intervals either side of the centre of the pulse so as to cover the region bounded by the ninth zeros on either side; the corresponding spread function of the lens considered was also sampled at 5 μ intervals. The two functions were then convolved by overlapping the curves, summing the products of coinciding samples, moving one curve relative to the other by 5 μ, repeating the summation, and so on. Thus a series of values representing the response of the lens to the (sin x)/x pulse was obtained; this series was then normalized so as to make the value corresponding to the peak of the reproduced pulse equal to unity and the values appropriate to the positions of the zeros of the (sin x)/x pulse were noted.

The calculations were made using a computer, the programme having been checked using a hand calculation.

3.1.2. The Spectrum Sampling Method

The spectrum of the pulse resulting from transmission by the lens of a (sin x)/x distribution is equal to the product of the spectrum of the (sin x)/x distribution and the optical transfer-function of the lens; it therefore consists simply of the lens response as far as the cut-off frequency and zero thereafter. The corresponding spatial response is the Fourier transform of this spectrum and may be obtained by multiplying the real component by cosines and the imaginary component by sines in the same way that the optical transfer-function of the lens was originally obtained from the spread function. In the computations made in this preliminary investigation the imaginary component was of very small amplitude and was therefore ignored.

The optical transfer-function was sampled at 15 points between zero and the cut-off frequency and the samples were multiplied by cosines in such a way as to obtain the response to a (sin x)/x distribution, measured at the zeros of the (sin x)/x.

Here a problem was observed. If the spectrum is sampled at spacing A, a spread function is developed centred at the origin which is contaminated by a similar one centred at $^1/A$ from the origin and another at $^2/A$, etc. The K-rating method requires a fairly accurate knowledge of the spread function at the positions of the (sin x)/x zeros; thus, very little contamination can be tolerated. In order to check the accuracy of the method the number of samples was doubled so as to double the spacing between the wanted and spurious spread functions. In addition it was arranged that the final sample was taken at the mid-point of the discontinuity in the spectrum at the cut-off frequency; this ensured that the contamination which did occur was small at the points of interest. The results obtained in this way agreed very closely, however, with those obtained using fewer samples, and it was therefore considered that sufficient accuracy had been achieved.

3.1.3. The Approximate Curve Method

In the limit, where samples are infinitesimally spaced, the process outlined in Section 3.1.2 becomes one of integration, represented by:

$$f(x) = \frac{1}{2\pi}\int_{0}^{2\pi f_c} R(\omega)\,.\,\cos \omega x \, d\omega + \frac{1}{2\pi}\int_{0}^{2\pi f_c} I(\omega)\,.\,\sin \omega x \, d\omega$$

where $\omega = 2\pi f$, f being the spatial frequency in, say, c/mm.

where $R(\omega)$ and $I(\omega)$ are the real and imaginary parts of the optical transfer function.

$R(\omega)$ is an even function, and its positive and negative halves are usually parabolic in shape. In fact, within the frequency range 0 to f_c, a parabola passing through points defined by the curve at frequencies 0, $\frac{1}{2}f_c$, and f_c is usually a very good fit to the curve.* Such a parabola, of the form $R(\omega) = a\omega^2 + b\omega + c$, was therefore assumed. The integration was carried out and the values of the resulting spread function at the points of interest were derived; the contribution made by the imaginary part $I(\omega)$ was again ignored.

It was found that if the modulation transfer-function is normalized so as to make its value equal to unity at zero frequency and the values at $\frac{1}{2}f_c$ and f_c equal to p and q respectively, then the normalized spatial response to a band-limited impulse at the positions of the (sin x)/x zeros is given by the following:

$$B_1 = B_{-1} = \frac{12\,(1 - q)}{\pi^2\,(1 + 4p + 6q)} \qquad B_2 = B_{-2} = \frac{6\,(1 + q - 2p)}{\pi^2\,(1 + 4p + 6q)}$$

$$B_3 = B_{-3} = B_1/9 \qquad B_4 = B_{-4} = B_2/4$$

$$B_5 = B_{-5} = B_1/25 \qquad B_6 = B_{-6} = B_2/9$$

$$B_7 = B_{-7} = B_1/49 \qquad B_8 = B_{-8} = B_2/16$$

3.1.4. Results of Preliminary Computation

K-ratings were obtained from the responses worked out as described in Sections 3.1.1, 3.1.2, and 3.1.3 by application of the restrictions outlined in Section 2.2. Table 1 summarises the results obtained.

TABLE 1

LENS	K-RATINGS			L_S
	CONVOLUTION	SAMPLING	APPROX. CURVE	
30 °T	4·28	4·35	4·41	-1·99
30 °S	1·89	1·85	1·80	-0·94
near 30 °S	2·33	2·13	2·09	-1·08
complete	1·85	1·85	1·83	-0·94

---

* For a more accurate result, particularly where the optical transfer-function does not approximate to a parabola, it may be represented by two parabolae, one extending from 0 to $\frac{1}{2}f_c$ and the other from $\frac{1}{2}f_c$ to f_c.

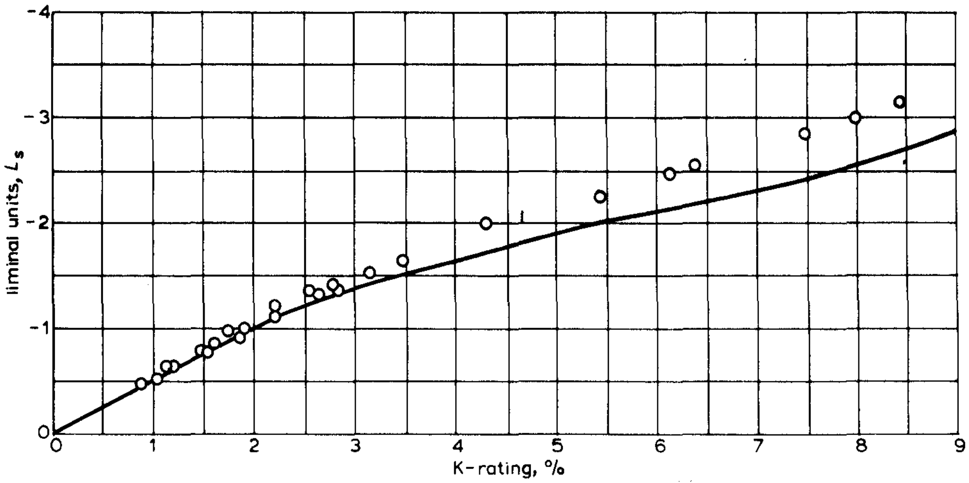

Fig. 4 - Tentative and empirically determined relationships between L_s and K-rating

—— tentative relationship shown in Fig. 2

o o o calculated relationships for specific cases

Agreement between the K-ratings obtained using the three methods is thought to be quite good, certainly good enough for practical purposes; the result obtained using the approximate-curve method agrees quite well considering that it uses only the magnitudes of the normalized transfer-function at two frequencies.

3.1.5. Final Computation

It was decided to use the convolution method* to determine the K-rating of the other 21 hypothetical lenses described in Section 3.1.

The K-ratings thus derived are shown as abscissae in Fig. 4, the ordinates being the values of L_S calculated from measurements of the frequency/response characteristics. Also shown is the tentative relationship illustrated in Fig. 3.

The agreement between the results is considered to be good, considering the limited accuracy of the information used to derive the tentative relationship.

3.2. The Theoretical Approach

Whilst the process outlined in Section 3.1 was being carried out it was noticed that the third of the four restrictions defined in Section 2.2 was in every case considerably more stringent than the others.

This characteristic was not unexpected. The main feature of the optical transfer-function of a lens is invariably a smooth increase in attenuation with increasing frequency. It has been shown[3] that narrow pulses transmitted by a device having a transfer function of this nature are mainly widened and their amplitudes reduced; in such circumstances the third restriction predominates.

In view of this, and since it was considered that a representative range of lens characteristics had been used in the analysis, it seemed reasonable to infer that K_3 would be the deciding factor for every lens. Having made this assumption, it was possible to derive a mathematical relationship between L_S and K-rating.

Fig. 5(*a*) shows the spectrum $F(j\Omega)$ and waveform $f(x)$ of a supposed $(\sin x)/x$ intensity distribution; $\Omega_c/2\pi$ is the cut-off frequency of the system.

The peak of the waveform is of magnitude

$$f(o) = \frac{1}{2\pi}\int_{-\Omega_c}^{\Omega_c} F(j\Omega)\,.\,d\Omega = \frac{\Omega_c}{\pi},$$

since the area under the spectrum is $2\Omega_c$.

The waveform passes through zero at intervals of π/Ω_c.

If the $(\sin x)/x$ distribution were transmitted by a lens, the spectrum $F_1(j\Omega)$ and waveform $f_1(x)$ of the image might typically be as shown in Fig. 5(*b*). The magnitude of this waveform at $x = 0$ is given by:

$$f_1(o) = b_o = \frac{1}{2\pi}\int_{-\Omega_c}^{\Omega_c} F_1(j\Omega)\,.\,d\Omega$$

which is now equal to $a\Omega_c/\pi$, where a is the area under $F_1(j\Omega)$ divided by the area under $F(j\Omega)$.

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* For convenience; a computer programme being available.

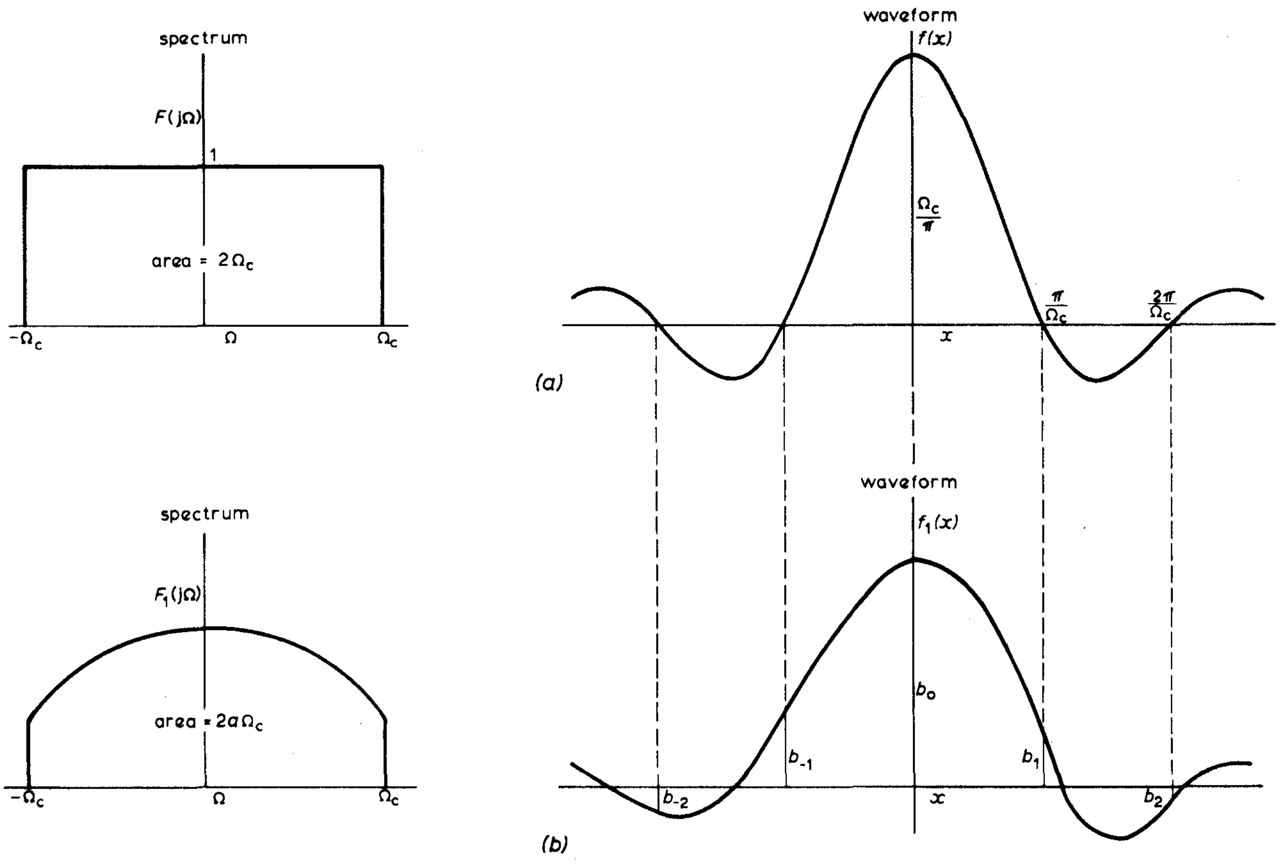


Fig. 5 - Spectrum and distribution of test pattern
(*a*) Before transmission by lens (*b*) After transmission by lens

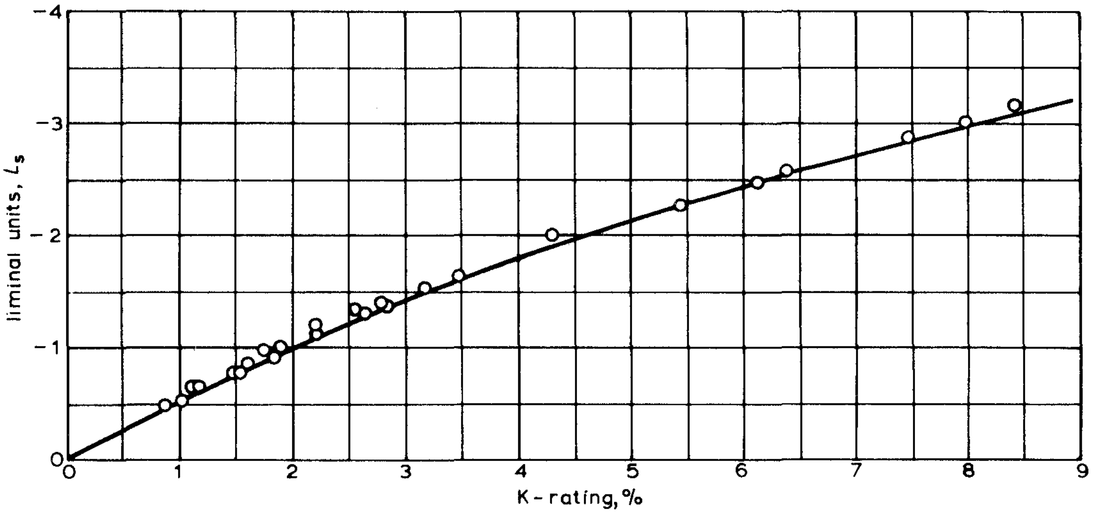


Fig. 6 - Relationships between L_s and K-rating derived by empirical and theoretical analysis
—— result of theoretical analysis ($K = K_3$)
o o o calculated relationships for specific cases

The magnitudes of the image waveform at points defined by the zeros of the (sin x)/x waveform will be:

$$b_1 = \frac{1}{2\pi}\int_{-\Omega_c}^{\Omega_c} \exp\left(\frac{j\Omega\pi}{\Omega_c}\right) F(j\Omega)\ .\ d\Omega$$

$$b_2 = \frac{1}{2\pi}\int_{-\Omega_c}^{\Omega_c} \exp\left(\frac{j2\Omega\pi}{\Omega_c}\right) F(j\Omega)\ .\ d\Omega$$

etc.

and so:

$$\sum_{-8}^{8} b = \frac{1}{2\pi}\int_{-\Omega_c}^{\Omega_c} \Big[1 + \exp\left(\frac{j\Omega\pi}{\Omega_c}\right) + \exp\left(\frac{-j\Omega\pi}{\Omega_c}\right) + \exp\left(\frac{j2\Omega\pi}{\Omega_c}\right) + \exp\left(\frac{-j2\Omega\pi}{\Omega_c}\right) + \ldots + \exp\left(\frac{j8\Omega\pi}{\Omega_c}\right) + \exp\left(\frac{-j8\Omega\pi}{\Omega_c}\right)\Big]\ F(j\Omega)\ .\ d\Omega$$

$$= \frac{1}{2\pi}\int_{-\Omega_c}^{\Omega_c} \Big[1 + 2\ \mathrm{Cos}\ \frac{\Omega\pi}{\Omega_c} + 2\ \mathrm{Cos}\ \frac{2\Omega\pi}{\Omega_c} + \ldots + 2\ \mathrm{Cos}\ \frac{8\Omega\pi}{\Omega_c}\Big]\ F(j\Omega)\ .\ d\Omega$$

It is shown in the Appendix that this is approximately equal to:

$$\frac{\Omega_c}{\pi}$$

Now $\Sigma_{-8}^{8}B_r$ used in the evaluation of K_3 is equal to $\Sigma_{-8}^{8}b_r$ normalized so that $b_0 = 1$. It is therefore equal to:

$$\frac{\Omega_c}{\pi} \cdot \frac{\pi}{a\Omega_c} = \frac{1}{a}$$

Therefore:

$$K_3 = \frac{1}{6}(\Sigma B_r - 1) = \frac{1}{6}\left(\frac{1}{a} - 1\right) \ldots\ldots\ldots (1)$$

Now the sharpness factor of the lens, as defined in Section 2.1 is equal to a, whence:

$$L_s = -9{\cdot}2\ (1 - a) \ldots\ldots\ldots\ldots\ldots (2)$$

We may now eliminate a from Equations (1) and (2), obtaining as a relationship between K_3 and L_s:

$$K_3 = \frac{-L_s}{6(L_s + 9{\cdot}2)} \ldots\ldots\ldots\ldots\ldots (3)$$

A graph of this relationship is shown in Fig. 6; the results of the experimental approach are also plotted in Fig. 6. It will be seen that the experimental results agree well with the theoretical relationship, having a tendency to give a slightly higher value of L_s for a given K-rating than Equation (3) would suggest. This is probably because, where the imaginary components of the optical transfer-functions were very small, they were ignored in the evaluations of L_s made in Section 3.1.

The assumption that K_3 is always the predominating K factor for lenses appears to be justifiable.[3] Furthermore if a lens could be found which had a K-rating not determined by K_3, the corresponding value of L_s (which has been shown to be related to K_3) would be an optimistic measure of subjective quality.

4. PRACTICAL IMPLICATIONS

It is normal practice for prototype models of the camera, telerecording, and scanner lenses used by the BBC to be measured in some detail, and Figures of Merit (combining L_s and L_v) are evaluated from the results of these measurements. The production models are not likely to have characteristics very different to those of the prototype unless they have been damaged or incorrectly assembled. Defects in a production lens produced in this way are almost invariably revealed if the results of a single on-axis measurement of the lens performance are compared with the results obtained from a similar measurement of the prototype. A lens testing bench has been installed at Television Centre for this purpose.

Routine measurements of lens performance are therefore unnecessary, and it is considered that the value of L_s (or of the corresponding K-rating) of the prototype lens is a sufficiently accurate measure of the resolution of production models, provided that these are not faulty.

Since the relationship between L_s and K-rating established in Section 3.2 exists, it is only necessary to compute one of these quantities, since Equation (3) or Fig. 6 then enables the other to be obtained.

It has been pointed out that the acceptance test by which the K-rating of electronic apparatus is established involves the use of a sine-squared pulse, with corrections to enable the response to a (sin x)/x pulse to be determined. This method could, with some difficulty, be applied to the K-rating of lenses; however, it would be simpler to use the square pulse produced by the slit, as currently used in sharpness measurements, together with the appropriate calculations. These calculations would in fact take the form outlined in Section 3.1.1. However, three factors discourage this procedure.

First, the process of obtaining a K-rating from spatial response measurements is very laborious, particularly if several spatial responses need to be combined to account for variations in response over the field. Secondly, even if the K-rating of a lens is determined, it may still be useful to know its optical transfer-functions so that these may be compared with the frequency/response characteristics of the other elements in the signal chain. Thirdly, modern lens testing benches[7] enable the optical transfer-functions to be obtained directly, so that the evaluation of L_S is relatively simple.

It is therefore recommended that values of L_S continue to be determined as at present, the corresponding K-ratings then being obtained, as required, by means of the relationship defined by Equation (3) and depicted in Fig. 6.

5. AN EXAMPLE OF THE USE OF THE FORMULA DETERMINED

It is of interest to use the formula relating L_S and K-rating to compare the BBC specifications defining the resolution required from television lenses and the K-ratings required from the video circuits of the associated cameras.

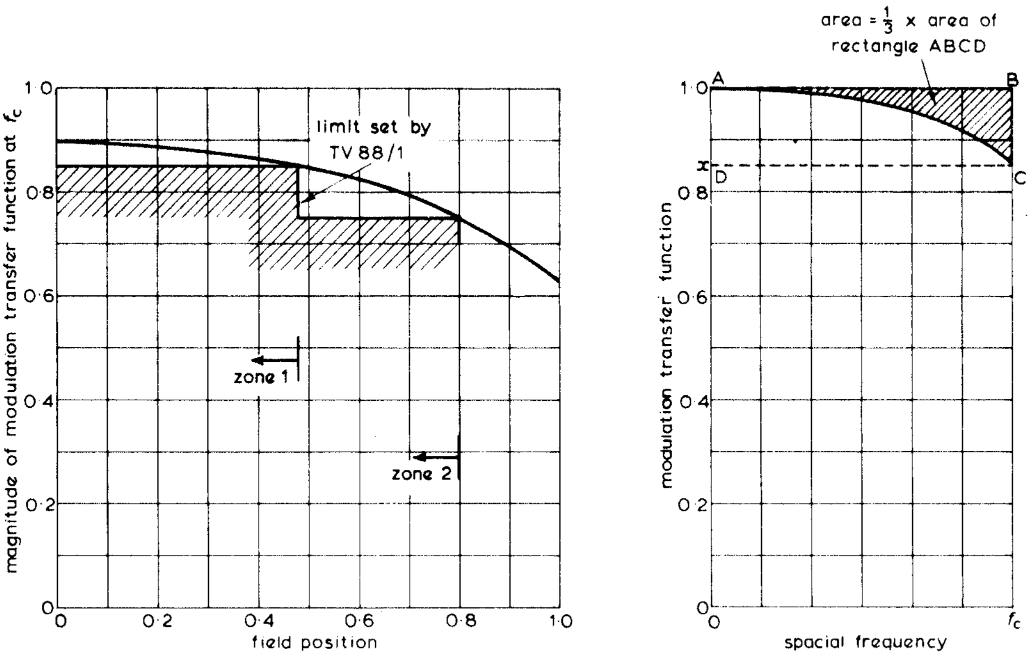
TV/88/1 specifies that fixed focus lenses should have modulation transfer functions whose magnitudes at the cut-off frequency f_c are not less than 0·85 at the edge of Zone 1 and not less than 0·75 at the edge of Zone 2. (These zones are defined by circles whose centres coincide with the centre of the field and whose radii are equal to 0·4 times the height and 0·5 times the width of the field respectively.)

Fig. 7(*a*) shows how the magnitude of the modulation transfer function at the cut-off frequency varies with field position for a typical lens just satisfying the above condition; the form of the modulation transfer function obtained at each field position is assumed to be as shown in Fig. 7(*b*) where each value of x is used to provide one ordinate of Fig. 7(*a*). Thus, the sharpness factor is given in each case by

$$\frac{2}{3}+\frac{x}{3}$$

When the weighting factors described in Section 2.1 have been applied, this leads to a sharpness index L_S of -0·51 limens. Using Equation (3) the corresponding K-rating is:

$$K = \frac{0{\cdot}51}{6(-0{\cdot}51 + 9{\cdot}2)} \times 100\%$$
$$= 0{\cdot}98\%$$



(*a*) (*b*)

Fig. 7 - Diagram outlining assumptions made in Section 5
(*a*) Assumed relationship between modulation at cut-off frequency and field position
(*b*) Assumed form of modulation transfer functions

Specification TV/139 lays down requirements for zoom lenses. If the above calculation is repeated using the values of x obtained from TV/139, a zoom lens just satisfying the specification is found to have a sharpness index varying typically from -0·63 limens at short focal length to -1·25 limens at long focal length. The corresponding K-ratings are 1·23% and 2·62% respectively.

It will now be appreciated that in terms of resolution, Specifications TV/88 and TV/139 for lenses impose a less stringent requirement than does Specification TV/142 for camera channels, since the latter are required to have an overall K-rating, excluding the lens and the tube, of less than 0·5%.

The resolution of the camera tube, however, is usually inferior to that of the other elements in the signal chain; this normally applies, for example, when fixed focus lenses are used with image orthicon tubes, or when zoom lenses are used with plumbicon tubes.

6. CONCLUSIONS

The relationship between the value of L_s for a lens and its corresponding K-rating is given by:

$$K = \frac{-L_s}{6(L_s + 9{\cdot}2)} \times 100\%$$

where L_s is expressed in liminal units, and K is expressed as a percentage in the normal way. It is recommended that this relationship be used to determine the K-rating of prototype lenses as required. The value obtained should then be a sufficiently accurate measure of the performance of production models. Thus the resolving properties of lenses may be compared with that of the other elements in the signal chain.

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APPENDIX*

Evaluation of an integral

We wish to solve:

$$\sum_{-8}^{8} b_r = \frac{1}{2\pi}\int_{-\Omega_c}^{\Omega_c}\left[1 + 2\,\mathrm{Cos}\,\frac{\Omega\pi}{\Omega_c} + 2\,\mathrm{Cos}\,\frac{2\Omega\pi}{\Omega_c} + \ldots\ldots + 2\,\mathrm{Cos}\,\frac{8\Omega\pi}{\Omega_c}\right] F(j\Omega)\,d\Omega$$

Note firstly that since b_{-r} is added to b_r, odd components cancel. We may therefore take for $F(j\Omega)$ the real part only of the optical transfer-function. Thus $F(j\Omega)$ is a continuous function which is even in Ω. Over the range $-\Omega_c$ and Ω_c it may be expressed by means of a Fourier series as:

$$F(j\Omega) = a_0 + a_1\,\mathrm{Cos}\,\frac{\Omega\pi}{\Omega_c} + a_2\,\mathrm{Cos}\,\frac{2\Omega\pi}{\Omega_c} + \ldots\ldots\infty$$

where $F(o) = 1$

$$= a_0 + a_1 + a_2 + a_3 + \ldots\ldots\ldots\ldots\infty$$

Moreover, since $F(j\Omega)$ is even:

$$\sum_{-8}^{8} b_r = \frac{1}{\pi}\int_{o}^{\Omega_c}\left[1 + 2\,\mathrm{Cos}\,\frac{\Omega\pi}{\Omega_c} + \ldots + 2\,\mathrm{Cos}\,\frac{8\Omega\pi}{\Omega_c}\right]\left[a_0 + a_1\,\mathrm{Cos}\,\frac{\Omega\pi}{\Omega_c} + a_2\,\mathrm{Cos}\,\frac{2\Omega\pi}{\Omega_c} + \ldots\right] d\Omega$$

Cross-product terms are of the form:

$$\int_{o}^{\Omega_c} A\,\mathrm{Cos}\,\frac{p\Omega\pi}{\Omega_c}\,\mathrm{Cos}\,\frac{q\Omega\pi}{\Omega_c}\,d\Omega = 0$$

and so we are left with terms of the form:

$$\int_{o}^{\Omega_c} A\,\mathrm{Cos}^2\frac{p\Omega\pi}{\Omega_c}\,d\Omega = A\,\Omega_c \qquad \text{if } p = 0$$

$$= \tfrac{1}{2}A\,\Omega_c \qquad \text{if } p \neq 0$$

$$\text{hence} \sum_{-8}^{8} b_r = \frac{\Omega_c}{\pi}\left[a_0 + a_1 + \ldots\ldots + a_8\right]$$

a_9, a_{10} etc are likely to be very small for an optical transfer-function, and it is therefore a very good approximation to take:

$$\sum_{-8}^{8} b_r = \frac{\Omega_c}{\pi}$$

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